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A Simplified Model of Midcourse Maneuver Execution Errors

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ABSTRACT

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Midcourse maneuvers are commonly employed in ballistic lunar and interplanetary space flight, and errors committed in executing these maneuvers contribute to target dispersion. A simplified model of such execution errors was developed at JPL. The model is presented in this Report, along with an expression for its second moment. AUTHOR

I. INTRODUCTION[1]

In ballistic lunar and interplanetary space flight, midcourse maneuvers are commonly employed in order to reduce dispersions caused by the launch vehicle (Ref. 1). The error committed in executing a midcourse maneuver will contribute, along with navigation errors, to the over-all target dispersion; hence a description of these "execution errors" is needed for accuracy analysis.

In this Report a simplified model of midcourse execution errors is given, and an expression for its second moment is presented. This model, which has been developed at the Jet Propulsion Laboratory (JPL), is especially appropriate for spacecraft such as *Ranger, Mariner*, and *Surveyor*, in which a complete inertial guidance system is not available, and in which maneuvers are performed by first commanding the spacecraft to assume a desired attitude and then commanding a desired velocity increment.

[1] Matrices are denoted by boldface letters, and vectors by italic letters with bars over them.

II. THE MODEL

Let $\bar{v}$ be the midcourse velocity maneuver that we desire to execute. We postulate that the execution error $\bar{e}$ is linearly composed of four independent errors, as follows:

1. **Shutoff Error.** The shutoff error $\bar{e}_s$ is in the direction of $\bar{v}$ and is proportional to $V \equiv |\bar{v}|$. This error would result from scale-factor errors in the shutoff system.

2. **Resolution Error.** The resolution error $\bar{e}_r$ is in the direction of $\bar{v}$ but is not dependent on its magnitude. Such an error would be caused by errors in computation and transmission.

3. **Pointing Error.** The pointing error $\bar{e}_p$ is perpendicular to $\bar{v}$ and proportional to V. Such an error would result from imperfect angular orientation of the thrust vector.

4. **Autopilot Error.** The autopilot error $\bar{e}_a$ is perpendicular to $\bar{v}$ and is not dependent on the magnitude V. This error can result from the behavior of the autopilot control system.

III. ANALYSIS

We next proceed to describe the execution errors mathematically and to develop their second moment. For the errors described above we shall find that the conditional probability $f(e|v)$ is Gaussian. The exact statistical nature of e is complex; it has often been found adequate to deal with the second moment of e, given by

$$\mathbf{L}_e = \mathrm{E}(\boldsymbol{e}\,\boldsymbol{e}^{\mathrm{T}}) = \overline{\boldsymbol{e}\,\boldsymbol{e}^{T}} = \int \boldsymbol{e}\,\boldsymbol{e}^{\mathrm{T}}\,\mathrm{f}(e)\,de$$

In the above expression $\boldsymbol{e}$ denotes a 3×1 column matrix representation of $\bar{e}$, the superscript T indicates transpose, and e is a three-dimensional variable. Since

$$\mathrm{f}(e) = \int \mathrm{f}(e|v)\,\mathrm{f}(v)\,dv$$

then

$$\mathbf{L}_e = \int\int \boldsymbol{e}\,\boldsymbol{e}^{\mathrm{T}}\,\mathrm{f}(e|v)\,\mathrm{f}(v)\,dv\,de$$

1. **Shutoff Error.** The shutoff error is given by $\bar{e}_s = s\bar{v}$, where s is a scalar random variable which is Gaussian $(0, \sigma_s)$. Then

$$\mathbf{L}_s = \mathrm{E}(\boldsymbol{e}_s\,\boldsymbol{e}_s^{\mathrm{T}}) = \mathrm{E}(s^2\,\boldsymbol{v}\,\boldsymbol{v}^{\mathrm{T}})$$
$$= \sigma_s^2\,\mathbf{L}_v$$

where $\mathbf{L}_v$ is the covariance of $\boldsymbol{v}$.

2. **Resolution Error.** The resolution error is given by $\bar{e}_r = r\,\bar{v}/V$, where r is a scalar random variable which is Gaussian $(0,\sigma_r)$.

Then

$$\mathbf{L}_r = \mathrm{E}(\boldsymbol{e}_r\,\boldsymbol{e}_r^{\mathrm{T}}) = \sigma_r^2\,\mathbf{G}$$

where

$$\mathbf{G} = \mathrm{E}\left(\frac{\boldsymbol{v}\,\boldsymbol{v}^{\mathrm{T}}}{V^2}\right) = \left\{\mathrm{E}\left(\frac{v_i\,v_j}{V^2}\right)\right\} \qquad i, j = 1,2,3$$

Thus $\mathbf{G}$ is the covariance of $\boldsymbol{v}/V$.

3. **Pointing Error.** Assume that $\bar{e}_p$ is circularly distributed in the plane perpendicular to $\bar{v}$. If $\boldsymbol{u} = (u_1, u_2, u_3)$ is a three-dimensional spherical Gaussian distribution in which

$$\mathrm{E}(u_i^2) = \sigma_p \qquad i = 1,2,3$$

then the cross product $\bar{u} \times \bar{v}$ is a proper representation for $\bar{e}_p$. Note that σ_p is in radians. Then

$$\bar{e}_p = \bar{u} \times \bar{v}$$

Writing out the components of $\bar{u}$ and $\bar{v}$, noting that

$$\mathrm{E}(u_i u_j) = 0 \qquad i \neq j$$

denoting

$$\overline{V^2} = \mathrm{E}(V^2) = \mathrm{E}(v_1^2 + v_2^2 + v_3^2)$$

and combining terms, yields

$$\mathbf{L}_p = \mathrm{E}(\boldsymbol{e}_p\,\boldsymbol{e}_p^{\mathrm{T}}) = \sigma_p^2\,(V^2\mathbf{I} - \mathbf{L}_v)$$

where $\mathbf{I}$ is the 3×3 unit matrix.

4. **Autopilot Error.** By an argument similar to the one above we obtain

$$\bar{e}_a = \bar{w} \times \frac{\bar{v}}{V}$$

where $\boldsymbol{w} = (w_1, w_2, w_3)$ is distributed similarly to $\boldsymbol{u}$, except that

$$\mathrm{E}(w_i^2) = \sigma_a \qquad i = 1,2,3$$

Note that σ_a is in units of velocity. Proceeding as before, we obtain

$$\mathbf{L}_a = \mathrm{E}(\boldsymbol{e}_a\,\boldsymbol{e}_a^{\mathrm{T}}) = \sigma_a^2\,(\mathbf{I} - \mathbf{G})$$

Finally, since

$$\mathbf{L}_e = \mathbf{L}_s + \mathbf{L}_r + \mathbf{L}_p + \mathbf{L}_a$$

we obtain

$$\mathbf{L}_e = (\sigma_s^2 - \sigma_p^2)\,\mathbf{L}_v + (\sigma_r^2 - \sigma_a^2)\,\mathbf{G} + (\sigma_p^2\,\overline{V^2} + \sigma_a^2)\,\mathbf{I}$$

It is interesting to note that if the proportional errors are equal, so that $\sigma_s = \sigma_p$, then insofar as a second-moment analysis is valid, the remaining contribution, namely $\sigma_p^2\,\overline{V^2}\,\mathbf{I}$, represents a spherical distribution of error. This can be easily checked; if $\sigma_s = \sigma_p$, the error is spherically distributed for any $\bar{v}$ and hence must be spherically distributed when averaged over all $\bar{v}$. A similar argument holds for resolution and autopilot errors. Finally, we note that if a knowledge of mechanization details for a spacecraft is not available, a spherical distribution for $\bar{e}$ appears to be the best assumption, even though $\bar{v}$ may have a highly preferred direction.

REFERENCE

1. Noton, A. R. M., Cutting, E., and Barnes, F. L., *Analysis of Radio-Command Mid-course Guidance,* Technical Report No. 32–28, Jet Propulsion Laboratory, Pasadena, California, September 8, 1960.